Bringing them all Together

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> [...] No way to say *warm* in French. There was only *hot* and *tepid*. If there's no word for it, how do you think about it? [...] Imagine, in Spanish having to assign a gender to every object: dog, table, tree, can-opener. Imagine, in Hungarian, not being able to assign a gender to anything: *he*, *she*, *it* all the same word. Thou art my friend, but you are my king; thus the distinctions of Elizabeth the First's English. But with some oriental languages, which all but dispense with gender and number, you are my friend, *you* are my parent, and YOU are my priest, and *YOU* are my king, and **You** are my servant, and ***You*** are my servant whom I'm going to fire tomorrow if ***You*** don't watch it, and **YOU** are my king whose policies I totally disagree with and have sawdust in **YOUR** head instead of brains, **YOUR** highness, and **YOU** may be my friend, but I'm still gonna smack **YOU** up side the head if **YOU** ever say that to me again: and who the hell are you anyway...?
>
> *Babel-17*
> Samuel R. Delany

1 What are Hybrid Logics?

Hybrid logics are modal logics and — at least, if the authors of this editorial had their way — vice-versa. Strictly speaking, not all modal logics are hybrid, but certainly any modal logics can be *hybridized*, and in our view many of them should be. What is it to hybridize a modal logic? To answer this properly we need to step back a little and discuss recent trends in modal logic.

Starting from the beginning, the new (sometimes call Amsterdam-style) perspective on modal logic, considers modal languages as general tools for talking about relational structures (for an up-to-date presentation of modal logic from this perspective, see [14]). A Kripke model for a propositional modal language is simply a set of points on which various relations have been defined, together with an assignment of atomic information — so Kripke models are just relational structures, the kinds of structures used to interpret first- and second-order classical languages. Relational structures are a useful tool in many disciplines: computer scientists can view labeled transition systems as relational structures, AI researchers can view various pictures of time as relational structures, description logicians can view networks of individuals as relational structures, and philosophers can view 'possible worlds' and the links between them in these terms. But none of these interpretations is privileged — and from the new perspective, that's the whole point. Relational structures are a fundamental modeling tool, and one reason why modal logic is so widely applicable is simply that it can be used to reason about whatever relational structures researchers find interesting.

But (according to the new perspective) there is a second reason why modal logic is so often the logic of choice. In essence, classical modal operators like $\Diamond$ and $\Box$ are 'macros' which help us uncover interesting fragments of first- and second-order classical logics. A unary diamond $\Diamond$ bundles a relatively simple form of classical quantification ("look for the information you are interested in at some accessible point") into an even simpler operator notation. The Until operator used in temporal applications bundles up a more complex $\exists\forall$ quantification pattern into a simple binary operator format. The $\langle\pi^*\rangle$ of Propositional Dynamic Logic ("look for the information you are interested by making a finite number of π transitions") bundles up quantification over the reflexive transitive closure of a relation into a unary operator. In short, the game of modal logic is about finding flexible and malleable operators which, when combined in different ways, yield well-behaved and useful fragments of classical logic.

The new perspective has had two positive effects. First, it has enriched our theoretical understanding of what modal logic is, for in order to fully understand these extensions (for example, to learn which fragments of classical logic they correspond too) new tools such as the Standard Translation and bisimulations are needed (see [14] for a full discussion of these concepts). Second, and just as importantly, it has encouraged modal logicians to think of themselves as 'logic engineers,' whose task is to craft logics to fit particular applications, and this has lead to the development of many new 'extended modal logics.'

It's at this point that hybrid logic comes in. Ordinary modal logics (even those with Until, or with the full apparatus of Propositional Dynamic Logic) have an obvious limitation: they lack a mechanism for referring to the points in models. And for many applications this is a genuine weakness: when reasoning about time, we often want to reason about what happens at a particular instant or interval, and when reasoning about terminologies, we often want to reason about how they apply to particular individuals. Ordinary modal logics don't deliver the goods here, and as logic engineers it is our job to investigate the situation, and add what is needed to complete the picture. This is the road that leads to hybrid logic, for one way to define "hybridization" is as the enrichment of ordinary modal logic with mechanisms for naming and reasoning about individual elements in the domain of a model.

2 Hybridizing Basic Modal Logic

To make the discussion concrete, let's see what is involved in hybridizing the basic modal language (that is, a propositional modal language containing only a single diamond $\Diamond$). Along the way we will meet many of the tools used by the contributors to this special issue: *nominals* (and *state variables*), *satisfaction operators*, the *global* *modality*, and *binders* (in particular, the $\downarrow$ *binder*).

Let's first recall the syntax and semantics of ordinary propositional modal logic. Given a set PROP of propositional variables p, q, r, and so on, we build formulas over PROP as follows:

$$\mathsf{WFF} \;:=\; p \mid \neg\varphi \mid \varphi \wedge \psi \mid \Diamond\varphi.$$

Other boolean connectives can be defined in the usual way, and $\Box\varphi$ is $\neg\Diamond\neg\varphi$.

Such a language is interpreted on models $M = (W, R, V)$. Here W is a non-empty set of points (or 'times,' or 'states,' or 'worlds,' or 'individuals,' depending on the application we have in mind), R is a binary relation on W, and V is a function with domain PROP and range 2^W (that is, V assigns to each propositional symbol the set of points at which it is true). The pair (W, R) is usually called a frame, and V is called a valuation. Given such a model, the satisfaction definition for our language is as follows:

$$\begin{array}{rll} M, w \models p & \text{iff} & w \in V(p) \\ M, w \models \neg\varphi & \text{iff} & M, w \not\models \varphi \\ M, w \models \varphi \wedge \psi & \text{iff} & M, w \models \varphi \;\&\; M, w \models \psi \\ M, w \models \Diamond\varphi & \text{iff} & \exists u(wRu \;\&\; M, u \models \varphi). \end{array}$$

If φ is evaluated at a point w in a model M, we say that w is the "point of evaluation" or the "current point." If $M, w \models \varphi$, we say φ is satisfied at M in w. If $(W, R, V), w \models \varphi$ satisfies φ for any choice of V or w, we say φ is valid on the frame (W, R).

Nominals and Satisfaction Operators

As the satisfaction definition just given makes clear, while $\Diamond$ is an elegant tool for quantifying over R-accessible points, the basic modal language offers us no tools for naming or reasoning about the points in W. Let's put this right by defining the *basic hybrid language*.

Let NOM be a set distinct from PROP. The elements of NOM are called *nominals* and are typically written i, j, k, and so on. We build formulas over NOM and PROP as follows:

$$\mathsf{WFF} \;:= i \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid \Diamond\varphi \mid @_i\varphi.$$

Nominals are the principal hybrid mechanism for referring to points, thus they play the role played by terms in classical logic. But note: *nominals are formulas, not terms.* Further, note that nominals can occur as subscripts to the @ symbol. Such a combination — for example, $@_i$ — is called a *satisfaction operator.*

The interpretation of this language is straightforward. The key step is to redefine what we mean by a valuation. We say that a (hybrid) valuation is a function V with domain $\mathsf{NOM} \cup \mathsf{PROP}$ and range 2^W such that for all $i \in \mathsf{NOM}$, $V(i)$ is a *singleton* set. That is, whereas ordinary propositional variables can be true at any number of points in a model, nominals are true at precisely one point in any model. They 'name' this point by being true there and nowhere else. We often call the unique point in $V(i)$ the *denotation* of i.

With that in place, add now the following two clauses to the satisfaction definition:

$$\begin{array}{rll} M, w \models i & \text{iff} & w \text{ is the denotation of } i. \\ M, w \models @_i\varphi & \text{iff} & M, u \models \varphi, \text{ where } u \text{ is the denotation of } i. \end{array}$$

That is, nominals are true at a unique point in any model (namely their denotation), and the satisfaction operators $@_i$ shifts the point of evaluation to the denotation of i. So $@_i\varphi$ says: "φ is satisfied at the point named by i."

Note that the formula prefixed by a satisfaction operator can itself be a nominal. For example, $@_ij$ is a well formed formula, and it has a useful meaning: it asserts that the nominal j is true at the point named by i, or to put it more simply, that i and j name the same point. Thus satisfaction operators give us a *modal theory of state equality*. Moreover, note that the formula $@_i\Diamond j$ means that the point named by j is a successor of the point named by i, so satisfaction operators also give us a *modal theory of state succession*.

From a logic engineering perspective, it should be clear that hybridization has added something useful. For example, if we were working in a temporal application (that is, if we think of the set W as a set of times, and the relation R as the relation of temporal precedence) then the formula $@_i\varphi$ says that the information φ holds at the time named by i — in short, it performs the same role as James Allen's [1] **Holds** predicate, but in a modal language. And if we think of W as a set of individuals, and R as a binary role (that is, if we are reasoning about terminologies) then $@_i\varphi$ is what a description logician would call an A-Box assertion.

What is less obvious is that far from having damaged the underlying modal logic, the hybridization we have just witnessed has arguably improved its logical behaviour. To give four examples:

1. *More expressivity over frames.* Ordinary modal languages have some surprising weaknesses when it comes to expressing properties of frames. For example, in ordinary modal languages there is no formula which defines irreflexivity (that is, in ordinary modal logic there is no formula valid on precisely those frames where $\forall w \neg wRw$). But the hybrid formula $@_i\neg\Diamond i$ defines this properly. Similarly, neither antisymmetry, asymmetry, nor discreteness can be defined in ordinary modal logic, but they can all be defined in hybrid logic. See [22, 11] for further discussion.

2. *No computational cost.* Often, adding nominals and satisfaction operators does not raise the complexity of the satisfaction problem. For example, the satisfaction problem of the basic modal language is PSPACE complete, and we remain in PSPACE if we add nominals and satisfaction operators. And the satisfaction problem for Propositional Dynamic Logic is EXPTIME-complete, and we remain in EXPTIME is we add nominals, satisfaction operators, and even the global modality. See [4] for further discussion.

3. *General completeness results.* One of the oldest themes in hybrid logic is that hybridization leads to simpler and more general completeness results. In a nutshell, this is because the presence of nominals and satisfaction operators makes it possible to combine the first-order Henkin construction with the modal canonical model construction (see Chapter 7.3 of [14]). Although ordinary modal logic has general completeness results (notably the Sahlqvist Theorem), these are typically complex to state and difficult to prove. In hybrid logic the situation is far simpler: *any formula which contains no propositional variable is guaranteed to be complete with respect to the class of frames it defines.* For example, $@_i\neg\Diamond i$

defines irreflexivity, and it contains no propositional variables, only nominals. So if we add it as an axiom to a suitable base logic, it is guaranteed to be complete with respect to the class of irreflexive frames.

4. *Proof-theoretical simplicity.* Ordinary modal logics are hard to work with proof-theoretically, for in general there is no simple way to get at the information under the scope of a modality. Again, nominals and satisfaction operators remove this difficulty. For example, suppose we are carrying out a tableau proof and we know that $@_i\Diamond\varphi$. By introducing a new nominal (say j) onto the tableau, we can decompose this information into $@_i\Diamond j$ and $@_j\varphi$, thus pulling the φ out from under the scope of the $\Diamond$. In short: we can carry out tableau proofs by constructing a modal theory of state succession. Another way of looking at it is that we are using a form of labeled deduction (see [20]) — but the labeling apparatus used here (namely nominals and satisfaction operators) belongs to the hybrid object language, hence we have *internalized* labeled deduction. Two contributions to this special issue (namely Seligman's, and that of Areces, de Nivelle and de Rijke) explore the proof theoretical ramifications of hybridization in detail.

Summing up: from both a theoretical and a logic engineering perspective, hybridization has much to offer. But we haven't yet told you everything you need to know. Two more topics remain: the global modality and binders.

The Global Modality

The global modality A (often called the universal modality [25]) can be quickly dealt with: $A\varphi$ means "φ is true at all points in the model." Thus $\neg A\neg\varphi$, which is usually abbreviated to $E\varphi$, means "φ is true at some point in the model." The modality is useful for a number of purposes — for example, to enforce global constraints on terminological definitions. However the operator was first isolated in the hybrid logical tradition, and for a rather different reason: it can be used instead of satisfaction operators, for $A(i \rightarrow \varphi)$ and $E(i \wedge \varphi)$ mean exactly the same thing as $@_i\varphi$. To some extent it's a matter of taste which approach is adopted, though it's worth knowing that adding nominals and satisfaction operators to the basic modal language does not take us out of PSPACE (see [4]), whereas adding the global modality (even if we don't add nominals) leads to an EXPTIME-complete satisfaction problem (see [39]).

Binders

A great deal could be said about binding. The basic idea is this: nominals, although they are formulas, are rather like the constants of first-order languages. So why not make it possible to *bind* occurrences of nominals, thereby increasing the expressive power still further? In fact, at least two ways of binding have been broadly investigated in the hybrid literature, namely *local binding* with the $\downarrow$-binder, and *global binding* with $\exists$ and $\forall$.

Roughly speaking, $\downarrow$ binds a nominal to the point of evaluation. Actually, just as first-order logic draws a distinction between constants and variables, hybrid logic draws a distinction between *state variables* and nominals. Syntactically, state variables are just like nominals, except that they can be bound and nominals can't. So it would be more accurate to say that $\downarrow$ binds a state variable to the point of evaluation. For example, consider the following formula:

$$\downarrow x \neg \Diamond x.$$

This names the current state x, and then insists that it is not possible to make an R-transition to the state named x. This, of course, will be true precisely when the state at which we are evaluating is not R-related to itself. To put it another way, it is a formula which distinguishes reflexive from irreflexive points in *any* model. No formula in ordinary modal logic, or even ordinary modal logic enriched with both nominals and satisfaction operators, can draw this distinction. The expressive power of $\downarrow$ is fully classified in [5].

Once you're accustomed to the idea of binding, it's tempting to go the whole hog and use the classical quantifiers $\exists$ and $\forall$, resulting in formulas like the following:

$$\exists x @_x \Diamond x.$$

This says: "somewhere in the the model there is a point x, and at the point named x it is possible to make an R-transition to the point named x," or more simply "the model contains a reflexive point."

A number of different binders are used in this special issue. Seligman uses the $\exists$ and $\forall$ binders as part of his proof theoretical investigation, Marx uses $\downarrow$ in the setting of relational algebra, and van Eijk *et al.* make use of a novel form of binding to describe network topologies.

That completes our bird's-eye view of hybrid logic. As we hope is now clear, hybrid logic offers something that ordinary modal logic does not — and yet the 'fit' between modal and hybrid logic is so organic that it would be artificial to regard them as separate disciplines. To give what may be a useful analogy: just as the classical logician can move freely between first-order languages with and without equality, we believe users of modal logic can (and should) add, discard (or invent!) hybrid apparatus as the need arises.

3 Historical and Bibliographical Remarks

The previous section told you what hybrid logics are, but not where they came from. In fact, hybrid logic has been around a surprisingly long time, and here we'll sketch their history and draw attention to a number of papers and other resources which may be useful to readers of this special issue.

Hybrid logic was introduced by Arthur Prior, the inventor of temporal logic, in the mid 1960s, and it played a fundemental role in his philosophical work. Drawing on McTaggart's [29] distinction between conceiving of time in terms of the A-series of past, present and future and the B-series of earlier and later, Prior introduced two logical systems. The T-calculus was intended to capture the A-series perspective, and used the tense operators F and P and variables ranging over propositions; it's what we would today call basic temporal (or tense) logic. The U-calculus aimed to capture the properties of the B-series, and made use of variables ranging over instants of time; it's essentially what a contemporary modal logician would call the temporal correspondence language.

Now, Prior viewed the A-series conception as fundamental, and wanted to show that 'tensed talk' could express everything that the U-calculus could. Unfortunately, the T-calculus (ordinary tense logic) is obviously weaker than the U-calculus (the temporal correspondence language) and Prior was well aware of this. Hybridization was Prior's response. In Chapter V.6 of [34], he enriches the T-calculus with instant-variables, allows them to be bound by $\forall$ and $\exists$, and adds (a variant of) the global modality. He called this "the third grade of tense-logical involvement" in [35, Chapter XI] and showed that the resulting system was strong enough to capture the U-calculus. In short, hybridization cleared the barrier to Prior's philosophical program of establishing the priority of tensed talk.

The technical development of hybrid logic was initiated by Prior's then student Robert Bull. In [17], a paper published in 1970, Bull proves a completeness result for a hybrid logic containing nominals, the $\forall$ and $\exists$ binders, the universal modality, and also *path nominals*. These 'name' branches in tree-like models of time by being true at all and only the points on the branch, thus they pick out a possible 'course of events.' Bull included path nominals in his hybrid logic — a decade before branching time logics were studied in theoretical computer science — because of Prior's interest in non-deterministic models of time. Bull demonstrates the relevance of the Henkin construction to hybrid logic, notes the ease with which richer logics can be dealt with, and suggests a novel approach (via non-standard models of set theory) to completeness theory.

The subject then seems to have lain dormant for over a decade until hybrid languages were independently reinvented by a group of logicians from Sofia, Bulgaria. George Gargov, Solomon Passy and Tinko Tinchev were interested in obtaining neat axiomatizations of various operations in Propositional Dynamic Logic. The problem here is that while certain operations (for example, union of programs) are easy to capture (union simply requires the axiom $\langle\alpha \cup \beta\rangle p \leftrightarrow \langle\alpha\rangle p \vee \langle\beta\rangle p$), a simple axiomatization of operations such as intersection or complement call for extra expressive power. In [30] it is shown that the addition of nominals is enough to provide succinct and natural characterization of intersection and complement. Moreover, the addition of other kind of "constants" to the language permits the representation of notions like determinism and looping [23] to be captured relatively straightforwardly. In addition, the work of the Sofia school showed how nominals could also be used to simplify the construction of models during completeness proof [31]. We strongly recommend [32] to readers of this special issue: it's a classic investigation of hybrid logic and the results and techniques remain relevant to contemporary concerns.

Hybrid logic entered its current phase in the 1990s, when a new generation of logicians (many of whom are represented in this special issue) became involved. The research of this period has lead in many new directions, but one general theme stands out: the exploration of weaker languages. For example, while Robert Bull and the Sofia school had realised that Henkin arguments could be used to prove completeness, their approaches require the use of relatively powerful hybrid languages. Similarly, in the 1990s it was realized that binding did not have to mean classical binding with $\forall$ and $\exists$, and the $\downarrow$ binder was isolated. For early papers in this phase, see [22, 11].

For the Paste rule, which permits Henkin methods to be used in the basic hybrid language, see [16]. For early work on $\downarrow$ see [24, 15], and for the current state of play see [5]. Hybrid proof theory, from a variety of perspectives, has blossomed in recent years, and we draw the readers attention to [38, 41, 18, 19, 12]. For interpolation results see [5], and for computational complexity, see [4].

That pretty much concludes the historical sketch — but it's worth stressing that we have only discussed what might be called 'mainstream' hybrid logic. One of the most exciting recent developments is the amount of work in neighbouring fields which echoes key hybrid logical themes. For example, the brand of labeled deduction developed by Basin, Matthews and Vigano [8, 9], links naturally with recent hybrid proof theory. Polish work on the logic of information systems and rough sets has lead to the evolution of what are essentially hybrid logics; see, for example, Konikowska [28]. For something close to hybrid logic, but developed from the perspective of first-order modal logic, see Gabbay and Malod [21]. Certain feature logics used in computational linguistics turn out to be hybrid logics (see [10, 36]) and in view of recent developments on HPSG, this line of work is overdue for a revival. Perhaps most interesting of all, however, is the increasing interplay between hybrid logic and description logic. For a detailed treatment of this link, see [3, 7]; for a hybrid-logical 'spypoint' argument being applied in description logic, see [40]; and for a recent description logic paper that makes a fundemental contribution to our understanding of hybrid logic, see [37].

If we have whetted your appetite, and you wish to learn more, then a good starting point is the Hybrid Logic Homepage

`http://www.hylo.net`.

Here you will find many of the papers mentioned above, and much other information besides. Also available there is the "Hybrid Logic Manifesto"[13] which is probably the most accessible introduction to the field currently available.

One final remark: we've mentioned a lot of theory, but what about implementation? As yet there is relatively little on offer, but this situation should soon change. On the Hybrid Logic Home page you can find a preliminary prototype of a prover for the basic hybrid language. This was implemented by Aljoscha Burchardt and Stephan Walter at Computerlinguistik, University of Saarland, as an undergraduate programming project (supervised by Patrick Blackburn), and you can experiment with the result over the web. At the University of Amsterdam, Carlos Areces and Juan Heguiabehere are implementing the direct resolution method presented in this special issue. Finally, the description logic community looks set to offer some useful tools, as Patel-Schneider recently announced that the next version of DLP [33] will support full nominals.

4 The Hybrid Logics Workshops

This special issue on hybrid logics was born from the HyLo 2000 Workshop, held in Birmingham as part of the Twelfth European Summer School in Logic, Language and Information (ESSLLI).

A year previously, the first HyLo workshop, HyLo'99, was organized by Patrick Blackburn at Computerlinguistik, University of Saarland, Germany. This closed workshop had two aims: to bring together researchers in hybrid languages to present recent developments, and to discuss how best to stimulate interest in the subject. HyLo'99 was the first workshop solely devoted to hybrid logic, and it made it clear that hybrid logics were gaining a new lease of life. At that time hybrid logics were starting to build strong links with different fields, notably description logic and labeled deduction, and its growing maturity meant that it could start to work as a theoretical framework for them.

After the first HyLo we knew we wanted to export hybrid logics to a wider audience, as we believed there were interesting ideas that many could profit from. ESSLLI was the perfect framework for these plans, with its wide mixture of backgrounds, covering logic, linguistics and computer science, and its broad attendance ranging from Masters and PhD students to leading researchers in these fields.

HyLo 2000: Bringing them All Together. HyLo at ESSLLI was always going to be quite different from the HyLo'99. For a start, it was a five day event, instead of a one day meeting. In addition, we didn't want another gathering of specialists in the area: we wanted to draw in as many people, from as varied backgrounds as possible.

The result was a complex formula, but we believe a successful one. HyLo 2000 was a mixture of workshop discussion, technical expositions, and tutorial presentation. We built the program around the invited speakers, and tried to fill in the details needed to draw a complete picture of the field.

To our delight, over 50 people attended HyLo 2000, a number we hadn't anticipated. We were particularly pleased, when on the second to last day, Martin Prior, Arthur Prior's son, was able to attend. The workshop seems to have filled its aim of raising the profile of hybrid logic: the number of visits to the hybrid logic web site increased dramatically following HyLo 2000 (we recently reached the 2300 hits). And, last but not least, HyLo 2000 provided the opportunity for this special issue.

5 Hybrid Logic in this Special Issue

After HyLo 2000, an open call for papers was circulated. From the papers received, the following five were accepted for publication:

Jerry Seligman. How to Create a Hybrid Calculus from its Semantic Theory. In this paper, Jerry Seligman takes us on an interesting journey. The satisfaction definition of most modal operators is specified in terms of first-order conditions. Hence we can always obtain a complete calculus for the basic logic characterizing any collection of such operators by appealing to a calculus which is complete for the full first-order language. Seligman shows here that by making use of the expressiveness provided by the hybrid apparatus, we can, step by step, transform a first-order sequent calculus into an internalized sequent calculus specifically tailored for a particular hybrid fragment.

Maarten Marx. Relation Algebra with Binders. Maarten Marx proposes extending the classical language of relation algebras with variables denoting individual elements in the domain and the hybrid binder $\downarrow$. This extension boosts the expressive power of the language to full elementary expressivity: any first-order property of binary relations can be now characterized. The most important part of Marx's article is the examples he discusses. These provide new perspectives on both relation algebra and hybrid logic.

Rogier van Eijk, Frank de Boer, Wiebe van der Hoek and John-Jules Meyer. Modal Logic with Bounded Quantification over Worlds. This paper develops a rather different kind of hybrid logics, from a rather different perspective. Driven by application issues (namely, to find the proper language to describe network topologies), van Eijk *et al.* arrive at a system which they describe as follows: "In comparison with standard hybrid languages, the logic covers separate mechanisms for navigation and for variable-binding and formalizes reasoning about the worlds of a model in terms of equational reasoning."

Carlos Areces, Hans de Nivelle and Maarten de Rijke. Direct Modal, Description and Hybrid Resolution. One of the reasons for hybridizing a modal logic is to try to improve its computational behavior. For example, as is discussed in [12], hybridization of basic modal logics leads to internalized labeled deduction. In this article, Areces *et al.* show how the same hybridization technique leads to simple resolution algorithms for modal and description logics. Going in the other direction, the use of resolution as a decision method for hybrid logics (which requires the use of paramodulation) sheds light on the view which regards hybrid logics as classical modal logics plus modal theories of state equality and state succession.

Valentin Goranko and Dimiter Vakarelov. Sahlqvist Formulas in Hybrid Polyadic Modal Logic. Goranko and Vakarelov investigate Sahlqvist's Theorem in the framework of hybrid logics. Building on the approach first discussed by the authors in [26], they provide a general description of hybrid formulas characterizing first-order properties of frames. A particularly interesting case is that of *reversive* languages, closed under all 'inverses' of modalities, because the minimal valuations arising in the computation of the first-order equivalents of Sahlqvist formulas are definable in such languages. This makes the proof of first-order definability and canonicity of these formulas a relatively simple syntactic exercise.

6 Other New Directions in Hybrid Logic

The articles in this special issue provide a reasonably broad perspective on hybrid logic, but they don't cover everything. Indeed, a sign of the field's health is that it is becoming increasingly difficult to keep abreast of developments — a novel situation in what has historically been a small field.

New developments in hybrid logic often come about by seeing the links which bind hybrid logic with other fields (this is certainly the case with the work relating hybrid logic to description logic, feature logic, and labeled deduction). And the same message keeps coming though: viewing other fields with hybrid eyes can lead to novel insights. Reciprocally, from these interactions hybrid logic acquires new proof methods, new directions for further development, and interesting problems to solve. We close this editorial by mentioning three new lines of work which will provide mind food for willing logic engineers.

Hybridizing First-Order Modal Logic. As we mentioned above, very expressive hybrid logics add different kind of binding and quantification across points to the underlying modal logic. But in most previous work, the underlying modal logic has been *propositional*. What happens when *first-order* modal logic is hybridized instead?

For over two decades, first-order modal logic has been something of a modal backwater. It is technically difficult terrain: coming up with general axiomatizations is hard, the area is plagued with frame incompleteness, and Craig interpolation and Beth definability are known to fail in a wide range of circumstances.

It is becoming clear that hybridizing first-order modal logic can cure many of these ills. For example, in a recent paper, Areces, Blackburn and Marx [6], show that equipping first-order modal logic with $\downarrow$ and @ yields systems with the Craig interpolation property (and hence Beth definability too). This holds for the logic of any class of frames definable in the bounded fragment of first-order logic, irrespective of whether constant, expanding, contracting, or arbitrary domains are assumed.

Why does hybridization open the door to general results in first-order modal logic? As in the propositional case, in essence because hybridization provides a frame language in which modal theories of equality and state succession can be formulated, and this makes it possible to blend first-order Henkin techniques with the use of modal canonical models.

Hybrid Logics and the μ-Calculus. A recent paper by Sattler and Vardi [37] investigates the logical language obtained by adding nominals and the global modality to the modal μ-calculus (with converse operators). Sattler and Vardi establish an ExpTime upper bound on the complexity of the satisfiability problem, thereby demonstrating the existence (as they put it) of a new ExpTime "Queen" logic.

The point is this. The use of the μ-binder over a modal logic with converse is already a powerful ExpTime complete system. But viewed from the perspective of (say) description logic, two familiar expressive weaknesses are apparent: general claims can't be formulated and individuals cannot be named. Adding the global modality and nominals solves these weaknesses, hence (from a description logic perspective) their result shows that it is possible to fully blend T-Box and A-Box reasoning in a system that can draw on the full resources of the modal μ-calculus with converse, without leaving ExpTime. Their system is a true "Queen," in which a number of important description logics can be straightforwardly embedded.

Just as interesting as the result, however, is the proof. Ordinary modal logics have the *tree property*: that is, satisfiable formulas can be satisfied on tree-based models, as a simple unraveling argument shows. But hybrid logics don't have this property: unraveling can destroy the requirement that nominals name unique points of the model. In spite of this, hybrid logics (without binders) are robustly decidable, and Sattler and Vardi's proof, which makes use of tree automata techniques, goes a long way towards explaining why.

Hybrid Quantification in Real Time Logics. Stéphane Demri and Valentin Goranko have recently called our attention to an interesting connection between the hybrid binder $\downarrow$, and certain operators introduced in the real time logics of Alur and Henzinger [2, 27]. In their proposal for a temporal logic modeling real time state transition systems, Alur and Henzinger were lead to models where each state has an associated value (which can be thought of as their time of execution). They then argue that a "retrieval operator" $x.\varphi$ is enough to express most interesting properties of such systems. For example,

$$\Box x.(p \to \Diamond y.(q \wedge y \leq x + 5)),$$

expresses that it is always the case that each request p is eventually followed by a response q within 5 units of time. Notice that x is similar to $\downarrow$: it creates on the fly, a syntactic reference to some "actual" value. There is a difference: whereas $\downarrow$ creates a transitory name for the actual state of evaluation, $x.$ retrieves the actual value associated with the state. But the ideas are closely related, and it seems likely that results can be transferred between the two lines of work.

This work also connects with first-order hybrid logic because, as the example above shows, the systems of Alur and Henzinger have a predicate structure which let us create terms like $x + 5$ and atomic formulas like $y \leq x + 5$. So $x.$ could be seen as a hybrid binder working on the first-order domain of each point, instead than on the domain of points.

As these examples show, neither HyLo 2000 nor this special issue managed to bring them *all* together — but that's simply because there were too many of them, surely an excellent sign. Modal logic is finding its way more and more each day into other fields (hardware and software verification, computational linguistics, spatial reasoning, knowledge representation, ...), and each step leads to interesting new territory. Hybridization has an important role to play in this process, for it provides tools that can be justified on both theoretical and logic engineering grounds. Go ahead, have a look, and let us know what you think.

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