Reichenbach, Prior and Montague: A semantic get-together

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1 Introduction

What do you write on for the logician who has written (or edited a Handbook) on just about everything? Finding something new isn't an option. So in the hope of reviving pleasant memories, we've decided instead to present some material related to work that Dov did quite some time ago, in the 1970s.

Dov Gabbay is a pioneer in the development of multi-dimensional modal and tense logic for natural language semantics; a record of his work in this area can be found in his book *Investigations into Modal and Tense Logics, with Applications to Problems in Linguistics and Philosophy* [Gabbay, 1976]. Multidimensional tense logics can improve on ordinary tense logic for natural language semantics in a number of ways; the most important is that they (or some versions of them at least) make it possible to 'store' times so that they can later be referred back to. This is important because, as is explained below, temporal reference is ubiquitous in natural language.

There is another style of modal and tense logic which makes reference to times possible, namely hybrid logic. And as the paper [Blackburn, 1994] pointed out, even a relatively modest hybrid logic (namely *nominal tense logic*) makes it possible to combine the ideas of Hans Reichenbach (see [Reichenbach, 1947]), who emphasized the importance of temporal reference, with those of Arthur Prior (see [Prior, 1967]), who emphasized the importance of the internal perspective on temporal structure provided by modal and tense logics. So Reichenbach and Prior have already met. But the paper that brought them together used a propositional version of nominal tense logic. To give semantic definitions that can be used with real grammars, more powerful systems are needed.

The pioneer of the use of richer logics in natural language semantics was Richard Montague who (most famously in his paper "The Proper Treatment of Quantification in Ordinary English" [Montague, 1973]) showed that higher-order logic was a superb tool for semantic construction. The higher-order logic that Montague developed for this purpose was called $\mathcal{IL}$ (Intensional Logic) and it made use of Prior's tense operators. Hence Prior and Montague have met up too.

So why not bring all three together to Dov's birthday party? That's what this paper is about. We are going to hybridize Montague's $\mathcal{IL}$ in the simplest possible way (we'll simply add nominals) to form $\mathcal{NIL}$ (Nominal Intensional Logic). We'll then show that the resulting system is capable of assigning nominal tense logical representations, which capture the insights of both Reichenbach and Prior, in a compositional way.

2 Prior and Reichenbach

Tense logic is a simple form of modal logic used for reasoning about time. It was invented by Arthur Prior, who introduced the F and P modalities (meaning "at some Future time", and "at some Past time" respectively) and their respective duals G and H ("it is always Going to be the case", and "it always Has been the case"). Why did he do this? Because Prior viewed tensed talk as fundamental: we exist in time, and deal with temporal information from the inside. He felt that the internal perspective offered by modal logics — where we evaluate formulas *inside* models, at some particular point — made it an ideal tool for capturing the situated nature of our experience and the way we talk about it.

For example, suppose we represent the meaning of the present-tensed English sentence "Dov smiles" by the propositional symbol *dov-smile*. If we prefix this with the P operator we obtain P*dov-smile*, and this is true at a time t if and only if Dov does indeed smile at some time t' previous to t. This captures (part of) the meaning of the past-tensed English sentence "Dov smiled". Moreover, the syntactic relationship between "Dov smiles" and "Dov smiled" (which differ only in their tense inflection) is reminiscent of the syntactic relationship between *dov-smile* and P*dov-smile*.

This is an interesting observation, and Prior's insistence on the importance of the internal perspective offered by tense logic deserves to be taken seriously. However, Prior's insight, though useful, fails to take into account the importance of *temporal reference* (that is, reference to specific times) in the semantics of tense (and indeed, in other natural language constructions).

Let's return to our example. The sentence "Dov smiled" does *not* mean that at some completely unspecified past time Dov did in fact smile (which is the meaning the tense logical representation P*dov-smile* gives it). Rather, it means that at some *particular*, contextually determined, past time Dov did in fact smile. Prior's tense logical representations are interesting, and correct as far as they go, but they do not go far enough. Moreover, in orthodox tense logic there is no mechanism to enable them to go further.

Hans Reichenbach, on the other hand, viewed temporal reference as central to the semantics of tense in natural language. He distinguished tenses in terms of the reference they make to three temporal markers, namely what he called the *point of speech* (S), the *point of event* (E), and the *point of reference* (R). Now, much of what he says is compatible with Prior's views. For a start, point of speech is the time at which the sentence is uttered, and this concept is fundamental to the internal perspective of tense logic: it's simply the particular time at which we chose to evaluate a formula in a given model. The point of event is the time at which the eventuality the sentence is talking about takes place. This might be the same time as the point of speech, or to its past, or to its future. This concept also fits naturally with Prior's tense logic. If φ is the representation of some eventuality, then evaluating φ at some time amounts to identifying point of event with point of speech. Prefixing P to form $\mathsf{P}\varphi$ locates the point of event to the past of the point of speech. Prefixing F to form $\mathsf{F}\varphi$ locates the point of event to the future of the point of speech.

However, Reichenbach's key innovation was the point of reference, and here we encounter something that orthodox tense logic cannot handle. Figure 1 tabulates Reichenbach's analyses of the tense forms of English. What do these analyses say?

Consider Reichenbach's account of the pluperfect. When we utter "I had seen", there is a clear intuition that we refer to some past time (this is the point of reference) and assert that the seeing event took place *before that.* Accordingly, Reichenbach analyzes the pluperfect form as E–R–S, which means that the point of event lies to the past of the point of reference, which in turn lies to the past of the point of speech.

What about the simple past? When we discussed "Dov smiled" we said that the function of the simple past in English was to locate an event at some particular (contextually determined) past time. In effect, this is what Reichenbach's treatment of the simple past gives us. He analyzes this tense as E,R–S. That is, the point of event and the point of reference coincide and lie to the past of the point of speech. The point of reference is the contextually determined past time, and by co-locating it with the point of event, we account for the way the simple past tense works.

Reichenbach's analyses are open to criticism. Some linguists have objected, for example, to the use of three distinct diagrams (namely R–E–S, R–E,S, and R–S–E) to account for the future-in-the-past tense (and indeed, the future perfect): as there is only a single natural language form, they demand a single representation. Moreover, many linguists would feel that his analysis of the present perfect (which amounts to saying that point of refer-

Structure	**Name**	**English example**
E–R–S	Pluperfect	I had seen
E,R–S	Past	I saw
R–E–S	Future-in-the-past	I would see
R–S,E	Future-in-the-past	I would see
R–S–E	Future-in-the-past	I would see
E–S,R	Perfect	I have seen
S,R,E	Present	I see
S,R–E	Prospective	I am going to see
S–E–R	Future perfect	I will have seen
S,E–R	Future perfect	I will have seen
E–S–R	Future perfect	I will have seen
S–R,E	Future	I will see
S–R–E	Future-in-the-future	(Latin: abiturus ero)

Figure 1. Reichenbach's referential analysis of tense

ence corresponds to point of speech) does not get to grips with the subtleties of this construction. Nonetheless, in spite of their shortcomings, Reichenbach's views on temporal semantics are highly influential in contemporary natural language semantics, and Reichenbach-inspired ideas lie at the heart of much recent work. This is because temporal reference in natural language is ubiquitous, and without some way of capturing its effects, we cannot adequately analyze many temporal constructions. Orthodox tense logic has fallen into disuse in natural language semantics largely because it offers no such mechanism.

3 Nominal Tense Logic

Prior didn't really like Reichenbach's ideas, but if you read the little he has to say on the subject (see pages 12–15 of [Prior, 1967]) you'll see that he doesn't offer any really solid criticisms. Rather, his dislike seems to stem from his conviction that what Reichenbach was saying was incompatible with a *logical* analysis of tense. This is ironic, since in the same book that he criticizes Reichenbach on these grounds, Prior also introduced a tool that allows his views to be integrated with Reichenbach's in a very smooth way! What was this tool? The key idea underlying modern hybrid logic: *sort* the propositional symbols, and use *terms as formulas*.

Let's see what this involves. Take a language of tense logic (with propositional symbols p, q, r, and so on) and add a second sort of propositional symbol. The new symbols are called *nominals*, and are typically written i, j, k, and l. Both types of propositional symbol can be freely combined to form complex formulas in the usual way. But the key ingredient is the following: *we insist that each nominal be true at exactly one time in any* *model*. Formally, the set of times used to interpret a nominal must be a *singleton* set. A nominal 'names' a time by being true at that single time and nowhere else.

This is a simple change, but it has important consequences. It immediately yields a more expressive logic. Consider the following (orthodox) tense logical formula:

$$\mathsf{F}(r \wedge p) \wedge \mathsf{F}(r \wedge q) \rightarrow \mathsf{F}(p \wedge q).$$

This can be falsified. The first conjunct in the antecedent says that in the future there is time where both r and p are true together, and the second asserts that in the future there is a time where r and q are true together. The conclusion then asserts that in the future there is a time where p and q are true together. But this is obviously unjustified: the future times that witness p and q may be distinct.

Now consider the following formula of nominal tense logic:

$$\mathsf{F}(i \wedge p) \wedge \mathsf{F}(i \wedge q) \rightarrow \mathsf{F}(p \wedge q).$$

This is identical to the preceding formula, except that we have replaced the propositional symbol r by the nominal i, but the resulting formula is impossible to falsify. We now have some extra information: the p-witnessing and q-witnessing future times both make i true, and there is only one time which does this, for i is a nominal. Hence these future times must be identical, and the conclusion follows.

However, what is important for present purposes is that in nominal tense logic it is possible to merge Prior and Reichenbach's ideas on tense. Prior never made the connection, but nominals are precisely the missing component needed to handle Reichenbach's points of reference! Figure 2 shows the table given earlier, but with nominal tense logical representations added in the final column.

Consider the representation $\mathsf{P}(i \wedge \mathsf{P}\varphi)$ of the pluperfect. This says that there is some time in the past labelled i and that the event φ happened *before that*. This representation combines Reichenbach's insight into the role played by temporal reference with Prior's insistence on the privileged role of tensed talk.

Note that in some cases the nominal tense logical representations improve on Reichenbach. In particular, note that the future-in-the-past (and the future perfect) now has a single representation. The formula $\mathsf{P}(i \wedge \mathsf{F}\varphi)$ asserts that there is a reference time i in the past, and that the point of event occurs to the future of i, which is what is wanted. We're not forced (as Reichenbach was) to spell out the irrelevant relationships that can hold between the point of event and the point of speech.

Structure	**Name**	**English example**	**Representation**
E–R–S	Pluperfect	I had seen	$\mathsf{P}(i \wedge \mathsf{P}\varphi)$
E,R–S	Past	I saw	$\mathsf{P}(i \wedge \varphi)$
R–E–S	Future-in-the-past	I would see	$\mathsf{P}(i \wedge \mathsf{F}\varphi)$
R–S,E	Future-in-the-past	I would see	$\mathsf{P}(i \wedge \mathsf{F}\varphi)$
R–S–E	Future-in-the-past	I would see	$\mathsf{P}(i \wedge \mathsf{F}\varphi)$
E–S,R	Perfect	I have seen	$\mathsf{P}\varphi$
S,R,E	Present	I see	φ
S,R–E	Prospective	I am going to see	$\mathsf{F}\varphi$
S–E–R	Future perfect	I will have seen	$\mathsf{F}(i \wedge \mathsf{P}\varphi)$
S,E–R	Future perfect	I will have seen	$\mathsf{F}(i \wedge \mathsf{P}\varphi)$
E–S–R	Future perfect	I will have seen	$\mathsf{F}(i \wedge \mathsf{P}\varphi)$
S–R,E	Future	I will see	$\mathsf{F}(i \wedge \varphi)$
S–R–E	Future-in-the-future	(Latin: abiturus ero)	$\mathsf{F}(i \wedge \mathsf{F}\varphi)$

Figure 2. Reichenbach's analysis in nominal tense logic

This is neat. However, as we pointed out at the start of the paper, the discussion has been conducted within the confines of *propositional* nominal tense logic. If we want to apply these ideas to real grammars for natural language, that's not good enough. And this leads us to the main topic of the paper: adding nominals to Montague's $\mathcal{IL}$.

4 Nominal Intensional Logic

In this section we take Montague's $\mathcal{IL}$ (which makes use of Prior's F and P operators) and add nominals to it. The result is called $\mathcal{NIL}$ (Nominal Intensional Logic). The following definitions follow Montague's treatment faithfully; the only deviations from his original work are the clauses we have added for handling nominals.

DEFINITION 1 (Syntax of $\mathcal{NIL}$). Let t, e, and s be any fixed objects. Then the set TYPES of types of $\mathcal{NIL}$ is defined recursively as follows:

$$\mathsf{TYPES} ::= t \mid e \mid \langle a, b\rangle \mid \langle s, a\rangle$$

where $a, b \in \mathsf{TYPES}$.

Basic Expressions: For each type a, $\mathcal{NIL}$ contains a denumerably infinite set of *non-logical constants* $c_{n,a}$, for each natural number n. The set of all non-logical constants of type a is called CON_a. For each type a, $\mathcal{NIL}$ contains a denumerably infinite set of *variables* $v_{n,a}$ for each natural number n. The set of all variables of type a is called VAR_a. Moreover, $\mathcal{NIL}$ contains a denumerably infinite set of *nominals* i_n for each natural number n. Nominals are of type t. The set of all nominals is called NOM.

Meaningful Expressions: The set ME_a of *meaningful expressions of type* a is defined recursively as:

1. Every variable of type a and every constant of type a are in ME_a.
2. Every nominal is in ME_t.
3. If $\alpha \in \mathsf{ME}_a$ and u is a variable of type b, then $\lambda u a \in \mathsf{ME}_{\langle b,a\rangle}$.
4. If $\alpha \in \mathsf{ME}_{\langle a,b\rangle}$ and $\beta \in \mathsf{ME}_a$ then $\alpha(\beta) \in \mathsf{ME}_b$.
5. If α and β are both in ME_a, then $\alpha = \beta \in \mathsf{ME}_t$.
6. If φ and ψ are in ME_t and u is a variable of any type, then the following are also in ME_t: $\neg\psi$, $(\varphi \wedge \psi)$ and $\exists u\varphi$.
7. If φ in ME_t, then $\Diamond\varphi$, $\mathsf{F}\varphi$ and $\mathsf{P}\varphi$ are in ME_t.
8. If $\alpha \in \mathsf{ME}_a$, then ${}^\wedge\alpha \in \mathsf{ME}_{\langle s,a\rangle}$.
9. If $\alpha \in \mathsf{ME}_{\langle s,a\rangle}$, then ${}^\vee\alpha \in \mathsf{ME}_a$.

DEFINITION 2 (Semantics of $\mathcal{NIL}$). A (standard) *model for* $\mathcal{NIL}$ is a 5-tuple $\langle A, W, T, <, F\rangle$ such that A, W and T are non-empty sets, $<$ is a linear ordering on the set T, and F is a function whose domain is the set of all non-logical constants of $\mathcal{NIL}$ together with the set of nominals NOM, F assigns to each non-logical constant a *sense* (as defined below). Moreover, for any nominal i, $F(i)$ must be a function with domain $W \times T$ and range $\{0,1\}$ such that for some unique $t \in T$, we have $[F(i)](\langle w,t\rangle) = 1$ (for all $w \in W$), and for all $t' \neq t$ (and all $w \in W$) we have $[F(i)](\langle w,t'\rangle) = 0$. That is, nominals denote functions that return 1 on one unique value of their temporal argument (the value of the world argument is irrelevant).

The set D_a of *possible denotations* of type a in a model $\langle A, W, T, <, F\rangle$ is defined as follows (where a and b are types):

$$\begin{aligned}
\mathsf{D}_e &= A.\\
\mathsf{D}_t &= \{0,1\}.\\
\mathsf{D}_{\langle a,b\rangle} &= \mathsf{D}_b^{\mathsf{D}_a}, \text{for } a \neq s\\
\mathsf{D}_{\langle s,a\rangle} &= \mathsf{D}_a^{W\times T}
\end{aligned}$$

The set S_a of *senses of type* a is defined as $\mathsf{D}_{\langle s,a\rangle}$. We can now complete the definition of the range of F in a model. The function F will assign to each non-logical constant of $\mathcal{NIL}$ of type a a member of S_a (we've already specified what it does with the nominals, and we can notice now that nominals just receive a special kind of sense in S_t).

An *assignment of values to variables* g is a function having as domain the set of all variables such that for any variable $v_{n,a}$, $g(v_{n,a}) \in \mathsf{D}_a$. We say that an assignment g' is a v-variant of an assignment g if it coincides with g in all values except perhaps in the value assigned to v.

Given a model $M = \langle A, W, T, <, F\rangle$, an assignment g, a world $w \in W$, and a time $t \in T$ we define, for any expression α the *extension of* α *with respect to model* M, w, t, g, denoted $[\![\alpha]\!]^{M,w,t,g}$, recursively as indicated in Figure 3.

5 Reichenbach and Prior via Montague

With the formalities out of the way, let's see how we can put $\mathcal{NIL}$ to work. The key tools we need are the following macros. These encapsulate in $\mathcal{NIL}$ the basic patterns needed for building nominal tense logical representations compositionally:

$$\begin{array}{rll} \mathsf{PAST} & =_{def} & \lambda V \lambda x^{\wedge}\mathsf{P}(i \wedge V(x)) \\ \mathsf{PLUPERF} & =_{def} & \lambda V \lambda x^{\wedge}\mathsf{P}(i \wedge \mathsf{P}V(x)) \\ \mathsf{PERF} & =_{def} & \lambda V \lambda x^{\wedge}(i \wedge \mathsf{P}V(x)) \\ \mathsf{FUTPAST} & =_{def} & \lambda V \lambda x^{\wedge}\mathsf{P}(i \wedge \mathsf{F}V(x)) \end{array}$$

Let's look at some examples. In what follows, we make use of standard Montague semantic representations. For example, the proper name "Dov" will have the representation

$$\mathsf{DOV} \quad =_{def} \ \lambda P^{\vee}(P\,dov),$$

and the semantic representation of "smile" will simply be SMILE. That is, what follows is classical Montague semantics, except that we make use of our Reichenbach-meets-Prior macros.

Here's a first example. Let's build a representation for "Dov smiled". We'll assume that this can be grammatically analyzed as (Dov(past(smile))). Hence we build its representation as follows:

$$\begin{array}{rll} \text{Dov} & \hookrightarrow & \mathsf{DOV} \\ & \hookrightarrow & \lambda P^{\vee}(P\,dov) \\ \text{smiled} & \hookrightarrow & \mathsf{PAST}(\mathsf{SMILE}) \\ & \hookrightarrow & \lambda V \lambda x^{\wedge}\mathsf{P}(i \wedge V(x))\mathit{smile} \\ & \hookrightarrow & \lambda x^{\wedge}\mathsf{P}(i \wedge \mathit{smile}(x)) \\ \text{Dov smiled} & \hookrightarrow & \mathsf{DOV}(\mathsf{PAST}(\mathsf{SMILE})) \\ & \hookrightarrow & \lambda P^{\vee}(P\,dov)\lambda x^{\wedge}\mathsf{P}(i \wedge \mathit{smile}(x)) \\ & \hookrightarrow & {}^{\vee}(\lambda x^{\wedge}\mathsf{P}(i \wedge \mathit{smile}(x))\,dov) \\ & \hookrightarrow & {}^{\vee\wedge}\mathsf{P}(i \wedge \mathit{smile}(dov)) \\ & \hookrightarrow & \mathsf{P}(i \wedge \mathit{smile}(dov)) \end{array}$$

$$
\begin{array}{rcl}
[\![c_{n,a}]\!]^{M,w,t,g} & = & [F(c_{n,a})](\langle w,t\rangle), \\
& & \quad \text{for } c_{n,a} \in \mathsf{CON}_a \\
[\![i_n]\!]^{M,w,t,g} & = & [F(i_n)](\langle w,t\rangle), \\
& & \quad \text{for } i_n \in \mathsf{NOM} \\
[\![v_{n,a}]\!]^{M,w,t,g} & = & g(\alpha), \\
& & \quad \text{for } v_{n,a} \in \mathsf{VAR}_a \\
[\![\lambda u\alpha]\!]^{M,w,t,g} & = & h, \text{where } h \text{ is the function with range } \mathsf{D}_b, \text{ s.t.} \\
& & \quad h(o) = [\![\alpha]\!]^{M,w,t,g'} \text{ for } g' = g[u \mapsto o] \\
& & \quad \text{for } u \text{ a variable of type } \mathsf{D}_b \text{ and } \alpha \in \mathsf{ME}_a \\
[\![\alpha(\beta)]\!]^{M,w,t,g} & = & [\![\alpha]\!]^{M,w,t,g}([\![\beta]\!]^{M,w,t,g}), \\
& & \quad \text{for } \alpha \in \mathsf{ME}_{\langle a,b\rangle} \text{ and } \beta \in \mathsf{ME}_a \\
[\![\alpha = \beta]\!]^{M,w,t,g} & = & 1 \text{ iff } [\![\alpha]\!]^{M,w,t,g} = [\![\beta]\!]^{M,w,t,g}, \\
& & \quad \text{for } \alpha, \beta \in \mathsf{ME}_t \\
[\![\neg\varphi]\!]^{M,w,t,g} & = & 1 \text{ iff } [\![\varphi]\!]^{M,w,t,g} = 0, \\
& & \quad \text{for } \alpha \in \mathsf{ME}_t \\
[\![(\varphi \wedge \psi)]\!]^{M,w,t,g} & = & 1 \text{ iff } [\![\varphi]\!]^{M,w,t,g} = 1 \text{ and } [\![\psi]\!]^{M,w,t,g} = 1, \\
& & \quad \text{for } \varphi, \psi \in \mathsf{ME}_t \\
[\![\exists u\varphi]\!]^{M,w,t,g} & = & 1 \text{ iff } [\![\varphi]\!]^{M,w,t,g'} = 1 \\
& & \quad \text{for some } u\text{-variant } g' \text{ of } g, \\
& & \quad \text{for } u \text{ a variable and } \varphi \in \mathsf{ME}_t \\
[\![\diamond\varphi]\!]^{M,w,t,g} & = & 1 \text{ iff } [\![\varphi]\!]^{M,w',t',g} = 1 \\
& & \quad \text{for some } w' \in W \text{ and } t' \in T, \\
& & \quad \text{for } \varphi \in \mathsf{ME}_t \\
[\![\mathsf{F}\varphi]\!]^{M,w,t,g} & = & 1 \text{ iff } [\![\varphi]\!]^{M,w,t',g} = 1 \\
& & \quad \text{for some } t' \in T, \text{ s.t., } t < t', \\
& & \quad \text{for } \varphi \in \mathsf{ME}_t \\
[\![\mathsf{P}\varphi]\!]^{M,w,t,g} & = & 1 \text{ iff } [\![\varphi]\!]^{M,w,t',g} = 1 \\
& & \quad \text{for some } t' \in T, \text{ s.t., } t' < t, \\
& & \quad \text{for } \varphi \in \mathsf{ME}_t \\
[\![{}^\wedge\alpha]\!]^{M,w,t,g} & = & h, \text{where } h \text{ is the function with range } W \times T, \text{ s.t.} \\
& & \quad h(\langle w',t'\rangle) = [\![\alpha]\!]^{M,w',t',g} \text{ for any } \langle w',t'\rangle \in W \times T, \\
& & \quad \text{for } \alpha \in \mathsf{ME}_a \\
[\![{}^\vee\alpha]\!]^{M,w,t,g} & = & [\![\alpha]\!]^{M,w,t,g}(\langle w,t\rangle), \\
& & \quad \text{for } \alpha \in \mathsf{ME}_{\langle s,a\rangle}
\end{array}
$$

Figure 3. Semantics of $\mathcal{NIL}$

This is the semantic representation given in our tables. Let's now build a representation for "Dov had smiled". We assume that this can be grammatically analyzed as (Dov(had(smile))). Hence we build its representation as follows:

$$
\begin{array}{rcl}
\text{Dov} & \hookrightarrow & \mathsf{DOV} \\
& \hookrightarrow & \lambda P^{\vee}(P\,dov) \\
\text{had} & \hookrightarrow & \mathsf{PLUPERF} \\
& \hookrightarrow & \lambda V \lambda x^{\wedge}\mathsf{P}(i \wedge \mathsf{P}V(x)) \\
\text{had smiled} & \hookrightarrow & \mathsf{PLUPERF(SMILE)} \\
& \hookrightarrow & \lambda V \lambda x^{\wedge}\mathsf{P}(i \wedge \mathsf{P}V(x))\mathit{smile} \\
& \hookrightarrow & \lambda x^{\wedge}\mathsf{P}(i \wedge \mathsf{P}\mathit{smile}(x)) \\
\text{Dov had smiled} & \hookrightarrow & \mathsf{DOV(PLUPERF(SMILE))} \\
& \hookrightarrow & \lambda P^{\vee}(P\,dov)\lambda x^{\wedge}\mathsf{P}(i \wedge \mathsf{P}\mathit{smile}(x)) \\
& \hookrightarrow & {}^{\vee}(\lambda x^{\wedge}\mathsf{P}(i \wedge \mathsf{P}\mathit{smile}(x))\,dov) \\
& \hookrightarrow & {}^{\vee\wedge}\mathsf{P}(i \wedge \mathsf{P}\mathit{smile}(dov)) \\
& \hookrightarrow & \mathsf{P}(i \wedge \mathsf{P}\mathit{smile}(dov))
\end{array}
$$

Again, this is the nominal tense logical representation we would expect from our earlier table. We leave the reader to experiment with other examples. The most important example to consider is, of course, "Dov will have smiled", but this is bound to work out right given that the birthday books are coming his way!

6 Conclusion

In this paper we have taken a first (short) step towards applying ideas from hybrid logic to natural language semantics. There's a lot left to do. For a start, we made use of only the basic tool of hybrid logics (namely nominals) but other tools (such at the @ operator) are clearly relevant to temporal semantics too (for example, @ can be used to help build up temporal representations for entire discourse, not just sentences). For further information on richer hybrid logics of time, see [Areces *et al.*, 2000]. Moreover, Montague's models are point based (that is, time is conceived of as a succession of instants), but for more detailed work it is useful to have access to temporal interval structure and to be able to evaluate formulas with respect to extended periods of time. It would be interesting to incorporate an interval based semantics to this system and add new modalities (such as a subinterval modality) to exploit it. Furthermore, it would be natural (following [Blackburn, 1994]) to shift to a two-dimensional pattern of evaluation to cover *temporal indexicals* such as "now", "yesterday", "today" and "tomorrow". The hybrid approach to these words makes use of the semantic machinery developed by Hans Kamp (see [Kamp, 1971]), Frank Vlach (see [Vlach, 1973]) and David Kaplan (see [Kaplan, 1977; Kaplan, 1989]), but exploits it using nominal-like propositional symbols *now*, *yesterday*, *today* and *tomorrow* rather than by adding new modalities. The move to a two-dimensional pattern of evaluation would also bring us one step nearer Dov's pioneering work.

But to conclude we want to note that there is a deeper sense in which this paper is linked to Dov's work: the simple fact that it uses hybrid logic. As far we are aware, Dov has never explicitly written on hybrid logic (though [Gabbay and Malod, 2002] comes close) but his work has influenced its development. In particular, his paper "An irreflexivity lemma" [Gabbay, 1981] and the development of labeled deduction (see [Gabbay, 1996]) provided ideas and insights that proved crucial to the development of hybrid deduction. More generally, via his writing Dov Gabbay has helped inspire key developments in modal (and many other kinds of) logic, and via his editorial work he has made it possible for logic to mature and come to grips with it relations with other fields. We are very happy to have this opportunity to pay our respects to Dov in this volume.

Happy birthday Dov!

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