A.N. Prior's Rediscovery of Tense Logic

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0. Introduction

In the field of logic, our century has seen a most striking rediscovery of the importance of time and tense. This is first and foremost due to the works of Arthur Norman Prior, who took a primary inspiration from his studies in ancient and medieval logic. In the 1950’s and 1960’s Prior laid out the foundation of tense logic, and showed that this important discipline was intimately connected with modal logic. He revived the medieval attempt at formulating a temporal logic for natural language. Prior also argued that temporal logic is fundamental for understanding and describing the world in which we live. He regarded tense and modal logic as particularly relevant to a number of important metaphysical problems: in particular, Prior analyzed the fundamental question of determinism versus freedom of choice by using the insights gained from the development of formal temporal logic.

Tense logic should not be seen merely as a new branch of logic like for instance deontic logic. In Prior’s opinion, logic in general should be understood as tense logic. He almost single-handedly introduced and formally developed this conception of logic, which revived ancient and medieval ideas regarding time and logic. For that reason Prior should be regarded as one of the greatest logicians of our century. The relevance of tense logic has now become clear not only to logicians but also to computer scientists and even to some physicists.
In this paper we intend to discuss: 1) The history of Prior's first formulation of ideas regarding tense logic in the early 1950's, 2) His idea of a new (or revived) approach to logic, 3) His use of tense logic in metaphysics, 4) His view on the relation between tense logic and natural science in general, 5) The conflict between the tensed view of time and the special theory of relativity.

The discussion introduces hitherto unknown or unnoticed aspects on Prior's work, stemming from an interview with Prior's widow, Dr. Mary Prior, and studies of unpublished papers kept at the Bodleian Library, Oxford. But our approach is not only historical. We also intend to clarify Prior findings. We shall argue that his main ideas regarding tense logic are very important, but also show that some of his positions in metaphysics can be critized.

1. Prior's Rediscovery of tense logic

Arthur Norman Prior was born in Masterton, New Zealand, on December 4th, 1914. His mother died a fortnight after his birth. His father was a doctor and a medical officer during the First World War, and Prior was brought up by his aunts and grandparents. Both of his grandfathers were Methodist ministers.

Prior went to Otago University at Dunedin in 1932. He set out to study medicine, but after a short time he instead went into philosophy and psychology. In 1934 he attended Findlay's courses on ethics and logic. Through Findlay Prior became interested in the history of logic and was introduced to Prantl's textbooks. His M.A. thesis was devoted to this subject. In 1949 Prior wrote about Findlay: "I owe to his teaching, directly or indirectly, all that I know of either Logic or Ethics" [Kenny p. 323].

Prior was brought up as a Methodist, but while he was a student he came to consider Methodistic theology too unsystematic, and he became a Presbyterian. He also became a very active member of the Student Christian Movement (SCM). In the years about 1940 he found himself in a crisis of belief. During these years he wrote the article "Can religion be discussed?" (1942), in which he advocated an almost atheistic position. This view, however, does not seem to have lasted very long. He continued to treasure his theological library and to join the work of the SCM [Kenny p. 326]. Later in his life, however, he became an agnostic.

In 1943 he married Mary. From 1946 to 1958 he taught philosophy at Canterbury University College in New Zealand. In 1953 he
became a professor of philosophy. In 1949 his book "Logic and the Basis of Ethics" had been published. After that time he became even more interested in logical problems. During 1950 and 1951 he wrote a manuscript for a book with the working title "The Craft of Logic". This book was, however, never published as a whole, but in 1976 P. T. Geach and A. J. P. Kenny edited parts of it. In the first chapter of the book, "Propositions and Sentences", the author among other things analyzed Aristotle’s view on some of the problems concerning time and tense. Prior found that according to the ancient as well as the medieval view a proposition may be true at one time and false at another. He described this view in the following way:

"... the statement or opinion that someone is sitting will be true so long as the person in question is in fact seated, and will become false - if it is persisted in - as soon as he rises."

[Prior 1976b, p. 38]

In the following years Prior worked mainly on questions in the history of logic. From 1952 to 1955 he had seven articles on the history of logic published. Four of these were concerned with Medieval logic and one with Diodorean logic. Hirst interest in the history of logic is also evident in his "Formal Logic", published in 1955. According to Mary Prior his resurging interest in the history of logic was very much due to the fact that the university library bought Bochenski’s "Précis de Logique Mathématique" (1948).

It seems that a short article by Benson Mates [1949] made Prior even more aware of the interesting relation between time and logic. The paper was concerned with Diodorean logic, primarily Diodorus’ definition of implication.

Prior seemed to realize that it might be possible to relate Diodorus’ ideas to contemporary works on modality by developing a calculus which included temporal operators analogous to the operators of modal logic. Mary Prior (in the afore-mentioned interview) recalled the first occurrence of this idea: "I remember his waking me one night, coming and sitting on my bed, and reading a footnote from John Findlay’s article on Time, and saying he thought one could make a formalized tense logic." This must have been some time in 1953 [Kenny p. 336]. The footnote which Prior studied that night was the following:

"And our conventions with regard to tenses are so well worked out that we have practically the materials in them for a formal calculus... The calculus of tenses should have been included in
the modern development of modal logics. It includes such obvious propositions as that
x present = (x present) present
x future = (x future) present = (x present) future;
also such comparatively recondite propositions as that (x). (x past) future; i.e. all events, past and future will be past." [Gale p. 159-60]

To be sure, Findlay’s considerations on the relation between time and logic in this footnote were not very elaborated, but it gave Prior the idea of developing a formal calculus which would capture this relation in detail. For this reason Prior called Findlay "the founding father of modern tense logic" [Prior 1967, p. 1]. But there are, in our opinion, certainly not sufficient reasons for viewing Findlay as the founder of tense logic. The honour of being the founder must without doubt be attributed to Prior himself. With his many articles and books on questions in tense logic he presented a very extensive and thorough corpus, which still forms the basis of tense logic as a discipline. Findlay’s major merit in tense logic is, as Jean-Louis Gardies [1975, p. 40] has remarked, to have had the luck of inspiring Prior to initiate the development of formal tense logic.

In fact, Findlay’s footnote was certainly not the only source of inspiration for Prior’s incipient formal study of the logic of time. Prior highly valued various parts of Polish logic like Lukasiewicz’s three-valued logic, and he was also acquainted with Reichenbach’s logical examinations of the tenses of verbs [Reichenbach 1947, p. 287-298]. And of course, from the previous stages of his career he was well acquainted with a huge historical material on questions related to temporal logic. A persistent feature throughout Prior’s works is a clear interest in the history of logic. Indeed, Prior took an interest in the history of logic not only as a subject in its own right, but he also saw the works of ancient and medieval logicians as a significant contribution to the contemporary development of logic. He was particularly interested in Aristotle, Diodorus, and the Scholastics, but his interest also extended to more recent logicians such as Boole and Peirce, whom he called "the greatest of all symbolic logicians" [1957c]. Relatively late he also became aware of McTaggart’s considerations concerning the A- and B-series conceptions of time [1908, p. 457-74].

Prior shared the medieval view on statements. He presented this view in "Past, Present and Future", quoting Peter Geach, who had formulated it as early as 1949:
"Such expressions as 'at time t' are out of place in expounding scholastic views of time and motion. For a scholastic, "Socrates is sitting" is a complete proposition, enuntiable which is sometimes true, sometimes false; not an incomplete expression requiring a further phrase like 'at time t' to make it into an assertion". [Prior 1967, p. 15]

Prior continued to examine the Scholastic sources himself, and in his writings he clearly demonstrated the validity of Geach's formulation of the Scholastics' view on propositions.

Prior was invited to Oxford as "John Locke Lecturer" in Philosophy in 1955-56. This led on to the Prior family moving in 1959 to Manchester and a few years later to Oxford, where Prior worked at Balliol College.

The John Locke lectures gave Prior an excellent opportunity to present his new findings regarding time and modality. The lectures were held on Mondays. Among the participants were John Lemmon, Ivo Thomas, and Peter Geach [Kenny p. 337]. The lectures were later published as the book "Time and Modality" [1957]. It was this work which made Prior internationally known. After the publication of "Time and Modality" he received a number of important and interesting letters from various logicians. One of the logicians who wrote to Prior was Saul Kripke. In two letters to Prior in September and October 1958 Kripke put forth some very stimulating ideas regarding temporal logic. The September letter contains an early version of the idea of branching time. Kripke suggested that we may consider the present as a point of "rank 1" and future possibilities at the next moment as points of "rank 2", and so forth. This gives rise to a tree structure representing the entire set of possible futures proliferating from the present. In this structure every point determines a subtree consisting of its own present and future.
Branching Time according to Saul Kripke, 1958:

Prior clearly found this view of time interesting, and he further developed the idea of branching time in his later writings. It is worth noting that the idea of time as a branching system had appeared in literary fiction before it was taken up by logicians and philosophers; the very same idea can already be found in a short story by Jorge Luis Borges first published in 1941, "The Garden of Forking Paths" [Borges, 1962, p. 89-101].

In his October letter Kripke brought another important question regarding tense logic to Prior’s attention. Kripke stated his doubt that tense logic is needed at all for scientific discourse. In Kripke’s opinion a tenseless logic might be preferable. Kripke referred to relativistic physics wherein two events may be simultaneous to one observer, but not to another. Thus one observer could truthfully say "It is now the case that A, and it is now the case that B", while another observer could truthfully say "It is now the case that A, and it is not now the case that B, although it will be the case that B." Prior realized the importance of this problem and it seems that
he often struggled with it. We shall deal with his attempted solutions in section 5.

In "Time and Modality" Prior had introduced the metric tense operators P(n) and F(n), meaning "it was the case n days ago that" and "it will be the case in n days that". He also studied the non-metrical operators, P and F, meaning "is has been the case that" and "it will be the case that".

In 1958 Prior entered into a very interesting correspondence with Charles Hamblin of The New South Wales University of Technology in Australia. Prior and Hamblin discussed two important issues in tense logic: the number of non-equivalent tenses, and the implicative structure of the non-metrical tense operators. In a letter to Prior dated 18th April 1958 Hamblin suggested a set of axioms with P and F as monadic operators, corresponding to "a simple interpretation in terms of a two-way infinite continuous time-scale".

Hamblin's axioms are:

Ax1: F(p ∨ q) ≡ (Fp ∨ Fq)
Ax2: ~F~p ⊃ Fp
Ax3: FFp ≡ Fp
Ax 4: FPp ≡ (p ∨ Fp ∨ Pp)
Ax5: ~F~pq ≡ (q ∨ Pq)

Hamblin also assumed 3 rules of inference:

R1: If A is a thesis, then ~F~A is also a thesis.
R2: If A ≡ B is a thesis, then FA ≡ FB is also a thesis.
R3: If A is a thesis, and A' is the result of simultaneously replacing each occurrence of F in A by P and each occurrence of P in A by F, then A' is also a thesis. (A' is the so-called mirror-image of A.)

When these axioms and rules are added to the usual propositional calculus a number of interesting theorems can be proved. In fact, Hamblin could prove that "there are just 30 distinct tenses", which can be formed using only P, F and negation. As a part of this result Hamblin demonstrated that any expression with more than two adjacent tense-operators could be reduced to one with two or less. This result can be summarized by the following diagram:
Hamblin also suggested a certain implicative structure for the tenses. His suggestion can be illustrated like this:

These results became even more appealing when Prior started to use the operators $G = \neg F\neg$ and $H = \neg P\neg$. We have not been able to find any explicit explanation as to why Prior chose exactly those two letters. However, M. J. Cresswell has by personal communication suggested to us that $G$ was inspired by the phrase "is always going to be", and $H$ by the phrase "has always been".

Finally, in 1965 Hamblin and Prior ended up with the following nice implicative structure for the non-metrical tense-operators, which according to Hamblin is "a bit like a bird’s nest" [Hamblin, letter of 6th July 1965]:

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In 1967 Prior published his major work, "Past, Present and Future", in which his approach to tense logic had reached a very convincing form. The decade of intense work in the field since the John Locke lectures had brought him a lot further. Also he had been able to benefit greatly from the correspondence with logicians like Kripke and Hamblin.

As a teacher Prior was very inspiring. He was always able to find nice and understandable illustrations of the logical systems he wanted to introduce. For instance, he would illustrate the fact that LMp cannot be deduced from Mp in the following way (where Mp in this context means "p either is or will be true", and Lp stands for "p is and always will be true"):

"it is or will be that Uncle Joe's car is running, but it will not always be true that this is or will be true; so in this sense Mp does not imply LMp" [1957c]

It seems clear that he very much liked teaching and lecturing. He was not "the Oxford type", but it appears that he almost immediately build up a reputation as one of the best lecturers in Oxford.

Prior died on October 6th., 1969, whilst on a lecture tour in Scandinaivia. On the day of his death he was visiting Trondheim in Norway. Prior had by then accomplished an impressive producti-
on. The bibliographical overview of Prior's philosophical works comprises more than 150 titles [Flo 1970]. In this overview one can follow how Prior's interests developed in the course of his work. Summarizing the main trends it can be said that his work until the middle of the 1950's was characterized by a preoccupation with ethics and the history of logic. From the mid-fifties and onwards he devoted himself mainly to the study of the relation between time, modality, and logic. That should be seen as a natural consequence of his endeavour to develop the formal calculus of tense logic, a task which he took up around 1953 (at the time of being inspired by Findlay's footnote). Nevertheless, we hope to have also made clear that there is no sharp distinction between Prior's philosophical and historical concerns on one hand and his work as a formal logician on the other.

2. A Tensed View on Time and Logic

According to Peter Geach, Prior regarded his own research into the logic of ordinary language constructions as a continuation of the medieval tradition [Geach p. 188]. His attitude was congenial to that of the young Russell in Principles of Mathematics: ordinary language is not a logician's master, but it must be his guide [Geach p. 187]. After all logic in Prior's opinion "is not primarily about language, but about the real world" [TR, p. 1]. For this reason he strongly opposed the formalistic view on logic:

"Formalism, i.e. the theory that logic is just about symbols and not about things, is false." [TR p. 1]
"I cannot see how any statement whatever can be made true simply by using language in a particular way..." [WL, p. 2]

Prior's own answer to the question about the nature of logic ran as follows:

"Logic deals, at bottom, with statements - it enquires into what statements follow from what - but logicians aren't entirely agreed as to what a statement is. Ancient and medieval logicians thought of a statement as something that can be true at one time and false at another." [SFTT, p. 1]

It is an obvious consequence of the ancient and medieval view that time should not be ignored in logic. Following this view Prior stressed that "the tense of a statement must be taken seriously" [SFTT, p. 2]. To Prior, all logic was in a sense tense logic: "...
tenseless statements of modern logic are just a special case of statements in the old sense ..." [SFTT, p. 2].

Prior argued that tense logic is based on two fundamental assumptions [Prior 1957a, p. 104]:
1) tense-distinctions are a proper subject of logical reflection,
2) what is true at one time is in many cases false at another time, and vice versa.

Prior observed that ancient and medieval logicians took these assumptions for granted, but that they were eventually denied (or simply ignored) after the Renaissance. Prior himself can be said to have realized the possibility of formulating a logic based on these old assumptions. In fact, he took the assumptions even further and also claimed the reality of tenses:

"So far, then, as I have anything that you could call a philosophical creed, its first article is this: I believe in the reality of the distinction between past, present, and future. I believe that what we see as a progress of events is a progress of events, a coming to pass of one thing after another, and not just a timeless tapestry with everything stuck there for good and all." [SFTT, p. 1]

Prior formally elaborated McTaggart’s distinction between an A- and a B-series conception of time [McTaggart 1908], and showed that we can discuss time using either a tense logic, or using an earlier-later calculus. Tense logic corresponds to the A-series conception, which sees time in terms of past, present, and future. An earlier-later calculus corresponds to the B-series conception, which sees time as a set of objectively existing instants. Prior clearly considered the A-conception to be the fundamental one:

"Time is not an object, but whatever is real exists and acts in time... But this earlier-later calculus is only a convenient but indirect way of expressing truths that are not really about ‘events’ but about things ..." [TR p. 2-3]

To Prior, tense logic was not merely a formal language together with rules for purely syntactic manipulations. It also embodied a crucial ontological and epistemological point of view:

"The tenses (it will be, it was the case) are primitive; only present objects exist." [Prior & Fine, 1977, p. 116]
Inspired by Quine's "Three Grades of Modal Involvement", Prior introduced four grades of "tense logical involvement". The first grade defines tenses entirely in terms of objective instants and the earlier-later relation. For instance, a sentence such as Fp, "it will be the case that p", is defined as a short-hand for "there exists some instant T which is later than now, and p is true at T". Tenses, then, can be considered as mere meta-linguistic abbreviations, so this is the lowest grade of tense logical involvement. Prior succinctly summarized the first grade as follows:

"... there is a nice economy about it ... it reduces the minimal tense logic to a by-product of the introduction of four definitions into an ordinary first-order theory, and richer [tense logical] systems to by-products of conditions imposed on a relation in that theory." [Prior 1968, p. 118]

In the first grade, tense operators are simply a handy way of summarizing the properties of the before-after relations, which constitute the B-theory of McTaggart. Hence, in the first grade B-theory concepts are seen to be determining for a proper understanding of time and reality; tenses are deemed to have no independent epistemological status. Moreover, instants acquire an independent ontological status. As we have seen, Prior rejected the idea of temporal instants as someting primitive and objective.

In the second grade of tense logical involvement, tenses are not reduced into B-series notions. Rather, they are treated on a par with the earlier-later relation. Specifically, a bare proposition p is treated as a syntactically full-fledged proposition, on a par with what Rescher & Urquhart [1971] called "chronologically definite" propositions such as T(a,p) ("it is true at time a that p"). The point of the second grade is that a bare proposition with no explicit temporal reference is not to be viewed as an incomplete proposition. One consequence of this is that an expression such as T(a,T(b,p)) is also well-formed, and of the same type as T(a,p) and p. Prior showed how such a system leads to a number of theses, which relates tense logic to the earlier-later calculus and vice versa. [Prior 1968, p. 119] The philosophical implication of this second grade of tense logical involvement is that one must regard the basic A- and B-theory concepts as being on the same conceptual level. Neither set of concepts is conditioned by the other.

The B-theory is sometimes considered as the semantics of the corresponding A-theory. This is not surprising if we again consider the first-grade formulation of Fp, "it will be the case that p", as a
short-hand for "there exists some instant T which is later than now, and p is true at T". In symbols:

\[ T(a, Fp) \equiv_{def} \exists b : a < b \land T(b, p) \]

This is tantamount to stating a truth condition for Fp. On this view of the relationship between the A- and B-theories, it may be a bit puzzling that p and T(a, p) can be treated as being on the same logical level - the former apparently belonging to the logical language (or object language) and the latter to the semantics (or meta-language). In Prior's opinion, however, this is not at all surprising. In a paper on some problems of self-reference he stated:

"In other words, a language can contain its own semantics, that is to say it own theory of meaning, provided that this semantics contains the law that for any sentence x, x means that x is true." [Prior (ed. Geach & Kenny) 1976a, p. 141]

It seems that this statement is exemplified exactly by the relation of the logic of tenses (the A-theory) to the logic of earlier and later (the B-theory), provided that we are willing to take the step of the second grade: syntactically conflating "bare" p with T(a, p). The relation becomes even clearer in the third grade, a system which has crucial implications for the status of the indication of time. Prior put it the following way:

"What I shall call the third grade of tense logical involvement consists in treating the instant-variables a, b, c, etc. as representing propositions." [Prior 1968, p. 122-23]

The traditional distinction between the description of the content and the indication of time for an event is thereby dissolved. From the properties of the logical language which embodies the third grade of tense logical involvement Prior also showed that T(a, p) can be defined in terms of a primitive necessity-operator. Then tense-logic, and indeed, all of temporal logic can be developed from the purely "modal notions" of past, present, future, and necessity. The primitive operators are F, P, and L. The entire earlier-later calculus is developed (one might say boot-strapped) from definitions in the tense-logical theory. In particular, "the instant a is earlier than the instant b", a < b, is defined by

\[ a < b \equiv_{def} L(a \supset Fb) \]

and the traditional semantical notion "it is true at time a that p" is defined by

\[ T(a, p) \equiv_{def} L(a \supset p) \]
(Recall that instants are equated with propositions.) In our opinion this idea of treating instants as some kind of world propositions was one of his most interesting constructions. We believe that the full strength of this view has not yet been displayed. It is very likely that this notion will turn out to be very useful in the part of computer science called natural language understanding [Hasle 1991].

The fourth grade consists in a tense logical definition of the necessity-operator such that the only primitive operators in the theory are the two tense logical ones: P and F. It appears that his reasons for wanting to reduce modality to tenses was mainly metaphysical, since it has to do with his rejection of the concept of the (one) true (but still unknown) future. As we shall see in the next section, his arguments for this view are interesting, but certainly not compelling.

Robin Le Poidevin [1991, p. 36 ff] has argued that Prior's propositional theory of instants is in tension with another basic tenet of Prior's, namely what Poidevin has called the anti-realist construal of past and future tensed statements. He has also maintained that the theory is based upon a incoherent view of propositions, namely the idea that different tokens of the same tensed type (e.g. 'Socrates is sitting') uttered at different times express the same proposition. In both cases Poidevin's criticism is based on the assumption that according to Prior's view tensed propositions have "present fact as their truth-conditions" [1991, p.37]. In our opinion, Poidevin's analysis is sound and interesting, but we believe that it based on a wrong assumption. In Prior's theory the notion of truth-conditions is not basic. It is in fact as we have just demonstrated a derived idea. The semantical concept of being true at an instant is defined in terms of present truth. But is Richard Swinburne [1990] has pointed out "there is more to be known about the world than you can know by knowing the truth-values of sentences at their time of utterance. You need to know which ones are true now, which of the ones which were, or will be true when uttered are true now. And for such truth timeless truth conditions cannot be given." [p.121] It is an essential element of Prior's theory that the very common assumption of truth as something timeless has to be rejected.

3. Tense Logic and Metaphysics
There can be no doubt that Prior’s view of time was strongly related to his commitment to what he called "a belief in real freedom". In his opinion, one of the most important differences between the past and the future is that "once something has become past, it is, as it were, out of our reach - once a thing has happened, nothing we can do can make it not to have happened. But the future is to some extent, even though it is only a very small extent, something we can make for ourselves..." [SFTT, p. 2]. Following Kripke’s ideas from 1958 Prior related his belief in real freedom to the concept of branching time:

"Genuine determinism would be the belief that there is only one possible future, and to express this you really do need to go beyond K_t and add a postulate for nonbranching of the future." [Prior 1969, p. 329]

(The system K_t - probably named in honor of Saul Kripke - is the calculus comprising "minimal tense logic", that is a tense logic with minimal restrictions on the structure of time.)

The postulate he has in mind is this one:  
\[
\text{PF}q \supset (q \lor \text{P}q \lor \text{F}q)
\]

"Whatever has been 'on the cards' either is the case or has been the case or is 'on the cards' still" [Prior 1969 p. 329].

As John P. Burgess [1978, p. 157] later explained, Prior would agree that the determinist sees time as a line, and the indeterminist sees it as a system of "forking paths". Prior incorporated this idea of branching paths into the concept of time itself. He found it highly important to examine the notion of branching in detail and proposed two models for it, inspired by Ockham and Peirce, respectively [Prior 1968, p. 122 ff.]. Nicholas Rescher is one of those who have reacted against Prior’s view on branching time. Rescher [1968] argues that time itself is not branching, although it must be admitted that a wealth of possibilities for the future course of events can be found. To Rescher, we have a "branching in time", but not "branching of time" [See Rescher & Urquhart 1971, p. 73 ff]. Storrs McCall [1976], on the other hand, has argued that our understanding of the passing of time is related to time as a branching system: the passing of time is equivalent to a loss of possibilities! The branching of time is thus thought to be branching in the "future direction" only, such that for any point in the system there exists only one possible past.
It is very likely that Prior's abandoning of Christianity and his becoming an agnostic was related to the problems concerning freedom and time. He was acutely aware of the fact that a number of significant Christian thinkers in the course of history had attacked or criticized the idea of free will. In a paper entitled "Determinism in Philosophy and Theology" [DPT] (probably written in his Calvinist period), he formulated this in the following way:

"It is extremely rare for philosophers to pay any great attention to the fact that a whole line of Christian thinkers, running from Augustine (to trace it no further back) through Luther and Calvin and Pascal to Barth and Brunner in our own day, have attacked free will in the name of religion." [DPT, p. 1]

Prior added that for instance Jonathan Edwards, who produced a novel defence of Calvinism in 18th-century New England, did it by "demonstrating the absurdity of free will itself" [DPT p. 1]. However, even if we accept that the idea of free will is illusory, the ordinary perception of freedom and of guilt have to be explained:

"Even those of us who accept a straightforward determinism have to give some account of men's feeling of freedom, and their feeling of guilt; ..." [DPT, p. 2-3]

"This state of inner conflict between two parts of the self, in which we feel both responsible and enslaved, is also one to which no one can be a stranger ..." [DPT, p. 4]

Even so, Prior felt that a Christian had to be a determinist, and that the believer must accept that we are guilty of that which we are totally helpless to alter [DPT, p. 2]. It appears that Prior accepted Edward's and others' argument that the doctrine of God's infallible and complete foreknowledge is incompatible with the contingency of future events. "I must confess I can't see that foreknowledge is compatible with preventability", he said [IWB, p. 12]. Prior clearly understood that foreknowledge should not itself be seen as the cause of that which is foreknown, but rather as an effect. But what has got so far as to have effects is surely "beyond stopping", he pointed out [IWB, p. 12]. The only way out of this for anyone who wants to accept the doctrine of divine foreknowledge appeared to be Thomas Aquinas' idea of atemporal knowledge. Thomas "taught that God doesn't experience time as passing, but has it present all at once. In other words, God sees time as tapestry" [SFTT, p. 2]. This solution was not at all attractive to Prior, since it seems to be in conflict with the reality of tenses. Moreover, atemporal
knowledge cannot be foreknowledge in the strict sense. Prior's own view was that God "cannot know the answer to the question "How will that person choose?" because there isn't any answer to it until he has chosen" [SFTT, p. 3].

Prior himself adopted the so-called Peirce-solution to the problem of the contingent future, according to which the following holds:

".. from the fact that there is a sea-battle going on it does not follow that there was going to be one, though it does follow that there will have been one." [Prior 1957a, p. 95]

Let q be the statement "there is a sea-battle going on". Then it is a thesis that

(*)  $q \supset F(n)P(n)q$

The validity of this thesis can be illustrated by the following diagram. If q is true to-day, then P(n)q will be true in any possible situation after n days. Therefore $F(n)P(n)q$ must be true to-day.

However, the corresponding statement (the mirror image of (*))

(#)  $q \supset P(n)F(n)q$

does not hold, if $F(n)q$ is understood in the strong way, i.e. as "it is bound to be the case after n time units that q" [Prior 1969, p. 329]. (This is the way it would have to be understood on the determinist view, which sees time as a single line with no branching.) The counter-argument against viewing (#) as a thesis is that it is possible to imagine that q is true at some time, but that $F(n)q$ has not been true n days ago, for which reason $P(n)F(n)q$ is not true now.
Similarly, Prior maintained that although
\[ q \supset Cq \]
is a thesis, the corresponding
\[ q \supset Hq \]
is not valid [SFTT, p. 2], if F is understood in the strong way. (It might, however, be accepted in a weaker sense, based on the notion of branching time: the truth of q entails the truth of Hq for some branch in the structure of time. If q is seen to be true w.r.t. the actual world, then the branch in question would be the one that had happened to be actualized in the course of history. That does not imply any determinism, for in this model nothing prevents that history might have taken another course.)

In Prior’s opinion, since the truth of future contingents cannot be known now, there cannot be any true statements about future contingents. On this view, the statement "there will be a sea-battle tomorrow" cannot be true today, since there is no unique future but rather a number of different possible futures. The Peircean solution to the problem of future contingents involves a certain vagueness. Prior quoted Peirce [1957a p. 114] on this:

"Again, statisticians can tell us pretty accurately how many people in the city of New York will commit suicide in the year after the next. None of these persons have at present any idea of doing such a thing, and it is very doubtful whether it can properly be said to be determinate now who they will be, although their number is approximately fixed.” [Peirce, Collected Papers, 4.172]
According to the Peircean solution the statement "Mr. Smith will decide to commit suicide in the year 1899" cannot be true in 1897, since Mr. Smith has not yet made up his mind at that time.

Prior was of course aware of a prominent alternative solution formulated in the Middle Ages. He carefully studied this medieval solution and referred to it as the Ockhamist solution. Consider again the main question concerning the interpretation of expressions regarding the future: Can it be maintained with conceptual and logical clarity that "(some event) E will happen", whilst viewing that statement as different from "E could happen" as well as "E will necessarily happen"? The medieval Ockhamist solution must be viewed as giving an affirmative answer to this question. According to Ockham's proposal, time can still be thought of as a branching structure, but with a unique though still unknown future among the possible futures:

In the foregoing we have tried to exemplify how there are two persistent yet apparently incompatible intuitions concerning the structure of time:

(1) We tend to believe that we can in fact make true or false statements about the future. We do not accept Prior's assumption that contingent propositions about the future cannot be true. We do not know of any good argument against the Ockhamistic idea that propositions about the future can be true even though their truth is presently not known by any human being. This means that we accept the idea of exactly one true future among the many possible futures:
(2) On the other hand, the passing of time can be viewed as a loss of possibilities, a view which implies a branching structure. That is, at any point of time there are several possible futures, even though all except one fail to materialize. This view is also compatible with our perception of free will.

The Ockham-system is an attempt at unifying these conceptions in a consistent way. Essentially, the Ockham solution embeds the notion of one actual future into a branching time structure. A crucial consequence of this move is that the truth now of a given proposition p does not in general imply the existence of other propositions, be they past or present, which entail the truth of p (now). Within the Ockham solution it must be accepted that a statement can be true without the present existence of any factual grounds for its truth.

4. Logic and the Natural Sciences

According to Prior logic is not merely about language. It is about the real world. For this reason it was important for Prior to discuss the relations between logic and the other sciences about the real world, such a the natural sciences. In some sense, Prior accepted the widespread view that logic is like a "parlour game", but he made an important comment to it:

"It is no accident that the best logician the University of Oxford has turned out, at all events since the Middle Ages is Lewis Carroll. But that does not make the subject any the less serious or important. With due respect to all the important people who ply the trade of atomic physics, I would venture to suggest that that at its best is a parlour game too." [1957b, p. 627]

Using logic we can deal with a number of different subjects and in each case we can attempt to create a formal symbol calculus. But it should be stressed that this is not always easily done. As Prior pointed out "some subjects do in fact have more order than others" [WL, p. 9]. Prior certainly saw it as profitable to construct a calculus for handling time and tense. He also suggested that a number of other fields could, at least to some degree, be treated using a formal calculus; for instance the logic of obligations, classical and relativistic physics, and some parts of economics. On the other hand, he very much doubted that a satisfactory formalization of biology could be developed [WL, p. 9-10]. Obviously, one of the
subjects with much order to it, and for which Prior believed a logic could be construed, was the theory of special relativity.

Prior realized that some physicists may say that tense logic is really not logic but physics, or that it has a great deal of physics "built into it". He admitted that this might be true, but he also pointed out that the borderline between logic and other subjects seems in any case not an easy one to draw except arbitrarily [Prior 1967, p. 51]. In consequence, it can never be very clear which part of tense logic is logic and which is physics. However, that does not constitute any crucial problem. After all the logician, as well as other scientists, wants to tell the truth about the world, and he is certainly ready to assist his fellow scientists. In Prior's opinion the logician must be "like a lawyer ... in the sense that he is there to give the metaphysician, perhaps even the physicist, the tense logic that he wants, provided that it be consistent. He must tell his client what the consequences of a given choice will be ... and what alternatives are open to him..." [Prior 1967, p. 59]

Prior's conception of time was clearly different from the view which is normally presented in textbooks not only in physics, but also in philosophy. In natural science the notion of events is fundamental, but according to Prior events do not "exist" at all; strictly speaking, only things exist. "Events are just what things do and what happens to them", he said [TR, p. 2]. Points of time, instants and events seemed as mythical to him as matter did to Berkeley [Prior 1967, p. 200]. In his view time is not an object, and the earlier-later calculus is just "a convenient but indirect way of expressing truths that are not really about 'events' but about things ..." [TR, p. 2-3].

It should also be mentioned that the idea of branching time is relevant not only to the theories of relativistic physics as well as the debate on determinism versus indeterminism. The connection to classical mechanics as well as quantum mechanics has also become evident as pointed out by C. F. von Weizäcker:

"The most general presupposition of experience is time. Its structure, as expressed by the words present, past and future, is analysed in a logic of temporal propositions (tense-logic) which provides the conceptual frame for quantum logic and for the theory of objective probability." [C. F. von Weizäcker 1971, p.236]
5. Tense Logic and Special Relativity

According to Prior many philosophers and scientists who accept the tapestry view of time have claimed that "they have on their side a very august scientific theory, the theory of relativity, and of course it wouldn't do for mere philosophers to question august scientific theories" [SFTT, p. 3]. Prior early became aware of the conflict between tense logic and special relativity. As we have seen in section 1, it was mention by Saul Kripke in a letter to Prior as early as 1958. Prior described the conflict in a very clear way:

"The trouble arises when we come to compare another's experiences, when, for example, I want to know whether I saw a certain flash of light before you did, or you saw it before I did. ...It could happen that if I assumed myself to be stationary and you moving, I'd get one result - say that I saw the flash first - and if you assumed that you were stationary and I moving, you'd get a different result ... And the conclusion drawn in the theory of relativity is that this question - the question as to which of us is right, which of us really saw it first - is a meaningless question ... Now I don't want to be disrespectful to people whose researches lie in other fields than my own, but I feel compelled to say that this just won't do." [SFTT, p. 3-4]

It is easy to understand what Prior means. Suppose that two observers, A and B, are moving with velocities v and -v, from an emitter E, both leaving E when the E-clock reads t=0.

```
  -v  A     E     B  v
     O   O     O
```

According to special relativity the following transformations for the time coordinates hold:

\[
\begin{align*}
  t_A &= L \left( t_E + vx_E \right) \\
  t_B &= L \left( t_E - vx_E \right)
\end{align*}
\]

where \( L = (1 - v^2)^{-1/2} \) and the speed of light is taken as unity (\( c = 1 \)).

A flash is emitted from E and received simultaneously by A and B, yielding same readings, \( t_E \), on the E-clocks. The time coordinates
for seeing the flash on A (\(x_E = -vt_E\)) and B (\(x_E = vt_E\)) can be calculated in A’s system in the following way:

\[
\begin{align*}
  t_{A,A} &= L(t_E + vx_E) = L(1-v^2)t_E \\
  t_{A,B} &= L(t_E - vx_E) = L(1+v^2)t_E
\end{align*}
\]

Clearly according to this A is the first to see the flash.

The arrivals of the light signals can also be calculated in the B-system:

\[
\begin{align*}
  t_{B,A} &= L(t_E - vx_E) = L(1+v^2)t_E \\
  t_{B,B} &= L(t_E + vx_E) = L(1-v^2)t_E
\end{align*}
\]

According to this calculation B sees the flash before A.

For this reason some physicists would say that the question as to which of the two observers really saw a certain flash first can only make sense if an inertial frame is specified relative to which the calculation should be carried out.

However, Prior thought that the question as to which of the two observers really saw a certain flash first is indeed a meaningful one. He stated that what it means is simply this: "When I was seeing the flash, had you already seen it, or had you not?" [SFTT, p. 5] Of course, it might be doubted that a physicist committed to the ordinary interpretation of special relativity would be convinced by that definition. He would probably say that this is begging the question. As a precondition for accepting the question as a meaningful one he would instead demand some experimental procedure, by means of which the question can be settled.

Prior admitted that we cannot in all cases know whether a given event is present or not, i.e. whether it is really taking place "now" or not, but he maintained that this epistemological question is very different from the corresponding ontological question. He stressed that all that physics might show is that "in some cases we can never know, we can never physically find out, whether something is actually happening or merely has happened or will happen." [Prior 1972, p. 323] Prior even called upon Albert Einstein’s authority in this matter, recounting that Einstein once said to Carnap that the problem of the Now worried him seriously. Einstein explained that "the experience of the Now means something special for men, something different from the past and the future, but that this important difference does not and cannot occur within physics". [Prior 1968, p.133-134] Following this kind of reasoning, Prior maintained that questions concerning the human Now make sense,
even though we cannot be sure that such questions can ever be decided by physical means. On logical and philosophical grounds Prior maintained that when an event X is happening, another event Y either has happened or has not happened. He strongly rejected the idea of treating "having happened" as a property that can attach to an event from one point of view whilst not from some other point of view:

"So it seems to me that there’s a strong case for just digging our heels in here and saying that, relativity or no relativity, if I say I saw a certain flash before you, and you say you saw it first, one of us is just wrong - is misled it may be, by the effect of speed on his instruments - even if there is just no physical means whatever of deciding which of us it is." [SFTT, p. 5]

There seems to be two different ways of solving the conflict between tense logic and special relativity. We can either reject (or adjust) the fundamental beliefs underlying tense logic, or we can reject (or adjust) the basic assumptions of special relativity.

A recent paper [Ohrstrøm 1990] analyzes a number of conceptual possibilities for upholding at the same time the assumptions of the Special Theory of Relativity and Prior’s equating reality with the present. The analysis shows that this can be done in various ways. One of the most obvious ways presupposes the selection of a privileged inertial system, to whose time-coordinates special meanings are attributed. If such a selection is not to be made ad hoc, then it must be possible to list the reasons (preferably cosmological ones) for it. It should be pointed out that the principle of relativity does not exclude a cosmological time (that is, a "natural" inertial system, which distinguishes itself through the distribution and movement of matter in the universe). However, his assumption of a homogeneous universe made him doubt whether cosmic time can actually be viewed as an ontological feature of the universe:

"It is doubtful whether there exists a precise definition which has so great merits that there would be sufficient reason to consider the time thus obtained as the true one." [Whitrow 1980, p. 304]

This point of view is not shared by all researchers, however. Gordon [1975], for example, thinks that a preferred system of inertia can actually be identified by cosmological investigations.

If this is so, Prior is right in claiming:
".. we may say that the theory of relativity isn't about real space and time... the time which enters into the so-called space-time of relativity theory ... is just part of an artificial framework which the scientists have constructed to link together observed facts in the simplest way possible..." [SFTT, p. 5]

Prior did not mind playing that parlour game, too. He realized that the non-linear structure of space-time points, ordered with absolute before-after relations, possibly of a causal nature, constitutes an interesting object of study for the tense logician. The structure branches both forwards and backwards, so it is not immediately clear how the corresponding tense logic is to be axiomatized. He argued [1967] that the characteristic axioms for relativistic space-time are:

\[
\begin{align*}
\mathrm{FGq} & \supset \mathrm{GFq} \quad \text{and} \\
\mathrm{PHq} & \supset \mathrm{HPq}
\end{align*}
\]

His argumentation was thorough and detailed, although a more systematic investigation of the relation between special relativity and tense logic was not carried out until 1980 (by Robert Goldblatt [1980]). A decade earlier on, Professor Gerald Massey from Michigan State University had directed a frontal attack on tense logic as a new discipline. He had specifically referred to results from the Special Theory of Relativity, accusing Prior of promoting "bad physics and indefensible metaphysics" [Massey 1969]. However, in the light of amongst other things Goldblatt's results, Massey's attack was somewhat unreasonable.

In fact Goldblatt could show that for any \( n \) there exists a well-formed formula \( f(n+1)p \), which is valid in the \( (m+1) \) dimensional Minkowski-space if and only if \( m < n \! \). This was certainly a remarkable result, but nevertheless Burgess in his overview of the status of tense logic had to observe that tense logic for special relativity had not yet been worked out fully, and that the results which had been seen so far had been extremely sparse [Burgess 1984, p. 130].

Regarding a tense logical approach to relativity, Prior also pointed out that there is a logic of such functors as "It appears from a certain point of view that -." Hence, it is possible to make good sense out of talk about an infinity of different "apparent" time-series. Prior suspected that the infinity of "local proper times", which figure in relativistic physics, amounts simply to what appears from various points of view, or what appears to be the course of events
in various "frames of reference". If the physicist wants to obtain a more general picture, he can "indicate what features of the course of events (what temporal orderings of those events) will be common to all points of view, and one can work out a tense logic for that too" [Prior 1968 p. 133]. Prior himself made some important contributions to the development of such a relativistic tense logic [Prior 1967, p. 203 ff.], although he felt that the relativistic project as a whole was a bit strange.

6. The Aim of Prior's Tense Logic - Concluding Remarks

The main reason why the relation between tense logic and the Special Theory of Relativity was important to Prior is related to the conceptual status of the present. To Prior, the present and the real were one and the same concept. Shortly before he died he formulated his view in the following way:

"...the present simply is the real considered in relation to two particular species of unreality, namely past and future." [Prior 1972]

Although Prior accepted the view that logic is like a parlour game, he also insisted that in a certain sense it is about reality. The logician's role in science and philosophy is to provide other scientists with sound logical systems. In his own words:

"... it may be that some day the mathematical physicists will want a sound logic of time and tenses; and meanwhile the logician had best go ahead and construct it, and abide his time." [SFTT, p. 6]

Today the logic of time and tense has found a main application in computer science. We have already briefly mentioned its use in program verification. It is also used in Artificial Intelligence for specification and as a knowledge representation language. In [Hasle 1991] it is demonstrated how one of Prior's tense logical systems, namely the afore-mentioned "third grade", gives rise to a natural logic programming implementation aimed at Natural Language Understanding. Temporal logics based on branching time have become important tools for program verification (see e.g. [Ben-Ari 1981; Thayse 1989]). At a still more fundamental level of computer science, temporal logic is now being developed as a programming language in its own right [Roger Hale 1987; Dov Gabbay 1987]. Thus, temporal logic seems to have a rather pervasive role
to play in computer science. A good overview is given in [Galton 1987].

Surprisingly, Prior himself in the 1960’s anticipated that there might be some practical gains from the study of tenses "in the representation of time-delay in computer circuits" and concerning an "accurate philosophical description of the reality of the passage of time" [TR p. 4]. The former remark would seem to anticipate its use in program verification with a special view to concurrency, while the latter may be relevant to its uses in AI and as a programming language. It would be in a good Priorean spirit to have logicians provide computer science with a collection of logics systems dealing with aspects of time, tense, and modality. One of the ideas which seems particularly fruitful for computer science, and which should be developed further, is the notion of branching time. This idea forms a basis for integrating tense logic with modal logic, i.e. for a tempo-modal logic. Branching time can be developed in at least two ways - a Peircean and an Ockhamistic.

In conclusion, we would like once more to emphasize that in dealing with questions of time and tense, Prior found much inspiration in the study of ancient and medieval logic. We think that this source of inspiration is still very useful as we have demonstrated elsewhere [Øhrstrøm 1984; Øhrstrøm & Hasle 1991].

Acknowledgements

The work contained in this paper has been carried out under a grant from the Danish Research Council for the Humanities within the Cognitive Science Research Programme. We are thankful to prof. M. J. Cresswell for providing useful information based on his experience as a former student of Prior’s. We are also grateful for the opportunity to study Prior’s unpublished material (papers and letters) now kept at the Bodleian Library in Oxford. Furthermore, Dr. Mary Prior has graciously given an interview, which has provided useful information on her late husband’s views and findings.

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